انتگرال[۲]
بخش ۲

۲ Sh. Nosrati, 13961208 update 14000126 ES0.

$$
\begin{aligned}
I_۶ &= \int \frac{\sqrt{x}}{۱-\sqrt{x}}\,\mathrm{dx} \\
&= \int \frac{\sqrt{x}}{۱-\sqrt{x}}\,\mathrm{dx} \quad ; \quad x = u^۲ \implies \mathrm{dx} = ۲u\,\mathrm{du} \\
&= \int \frac{۲u^۲}{۱-u}\,\mathrm{du} \\
&= \int -۲u - ۲ + \frac{۲}{۱-u}\,\mathrm{du} \\
&= -u^۲ - ۲u - ۲\ln(۱-u) + C \\
&= -x - ۲\sqrt{x} - ۲\ln(۱-\sqrt{x}) + C
\end{aligned}
$$

$$
\begin{aligned}
I_{۷} &= \int \frac{\sqrt{x}}{\sqrt{۱-x}}\,\mathrm{dx} \\
&= \int \frac{\sqrt{x}}{\sqrt{۱-x}}\,\mathrm{dx} \quad ; \quad \begin{cases} x = \sin^{۲} u \\ \mathrm{dx} = ۲\sin u \cos u \;\mathrm{du} \end{cases} \\
&= \int \frac{\sin u}{\cos u} ۲\sin u \cos u \;\mathrm{du} \\
&= \int ۲\sin^{۲} u \;\mathrm{du} \\
&= \int ۱ - \cos ۲u \;\mathrm{du} \\
&= u - \frac{۱}{۲}\sin ۲u + C \\
&= \arcsin\sqrt{x} - \sqrt{x(۱-x)} + C
\end{aligned}
$$

$$
\begin{aligned}
I_{۸} &= \int_{۰}^{۱} \frac{\sqrt{x}}{\sqrt{۱-x}}\,\mathrm{dx} \\
&= \left(\arcsin\sqrt{x} - \sqrt{x(۱-x)}\right)_{۰}^{۱} \\
&= \arcsin ۱ \\
&= \frac{\pi}{۲}
\end{aligned}
$$

$$\begin{aligned}
I_{۹} &= \int_{۰}^{۱} \frac{\sqrt{x}}{\sqrt{۱-x}}\,\mathrm{dx} \\
&= \int_{۰}^{۱} x^{\frac{۱}{۲}}(۱-x)^{-\frac{۱}{۲}}\,\mathrm{dx} \\
&= \beta\left(\frac{۳}{۲},\frac{۱}{۲}\right) \\
&= \frac{\Gamma\left(\frac{۳}{۲}\right)\Gamma\left(\frac{۱}{۲}\right)}{\Gamma\left(\frac{۳}{۲}+\frac{۱}{۲}\right)} \\
&= \frac{\Gamma(\frac{۳}{۲})\Gamma(\frac{۱}{۲})}{\Gamma(۲)} \\
&= \frac{\frac{۱}{۲}\sqrt{\pi}\sqrt{\pi}}{۱} \\
&= \frac{\pi}{۲}
\end{aligned}$$

$$
\begin{aligned}
I_{۱۰} &= \int \frac{x-\sqrt{x}}{x+\sqrt{x}}\,\mathrm{dx} \\
&= \int \frac{x-\sqrt{x}}{x+\sqrt{x}}\,\mathrm{dx} \quad ; \quad x = u^{۲} \implies \mathrm{dx} = ۲u\,\mathrm{du} \\
&= \int \frac{u^{۲}-u}{u^{۲}+u}۲u\,\mathrm{du} \\
&= \int ۲u - ۴ + \frac{۴}{u+۱}\,\mathrm{du} \\
&= u^{۲} - ۴u + ۴\ln(u+۱) + C \\
&= x - ۴\sqrt{x} + ۴\ln(\sqrt{x}+۱) + C
\end{aligned}
$$